Revisiting Metastable Cosmic String Breaking

Based on 2312.15662

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Collaboration with

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Outline

1. Metastable Cosmic String
   Motivation.
   Gravitational wave and NANOGrav result.

2. Metastable String with Monopole

3. Revisiting String Breaking

4. Summary

Topological Object in BSM

Extension of Standard Model can have larger symmetry.

$$G_{\rm BSM} \to G_{\rm SM} = {\rm SU}(3)_C \times {\rm SU}(2)_L \times {\rm U}(1)_Y$$

Example:

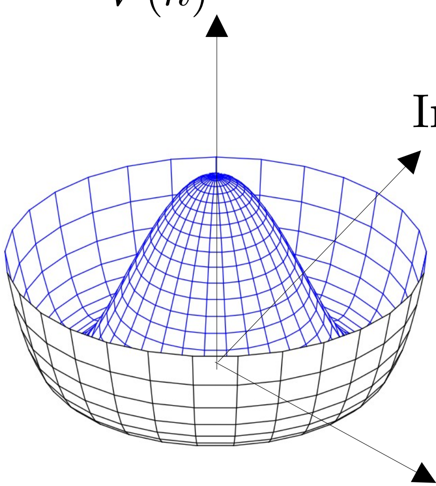
Grand Unified Theory, SU(5), SO(10)….: Monopole, cosmic string...

$U(1)_{PQ}$ extension for Axion: Domain wall, cosmic string.

Cosmic String

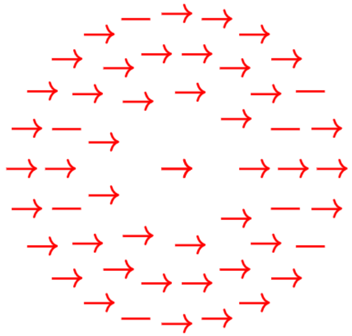
$$\mathrm{U}(1) \to \text{nothing}$$

$$\langle h \rangle \neq 0$$



Spontaneous
Symmetry
Breaking

Phase of h



Winding number 0

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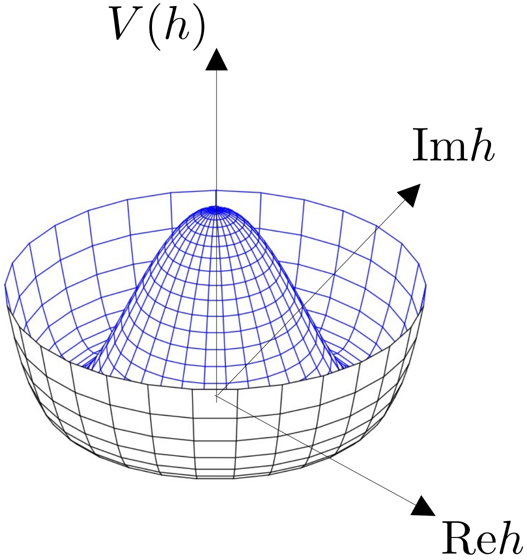

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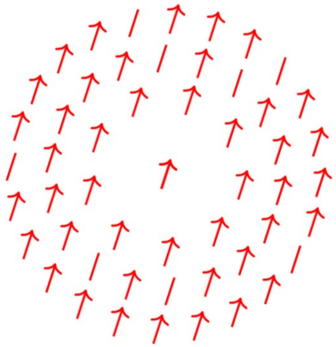

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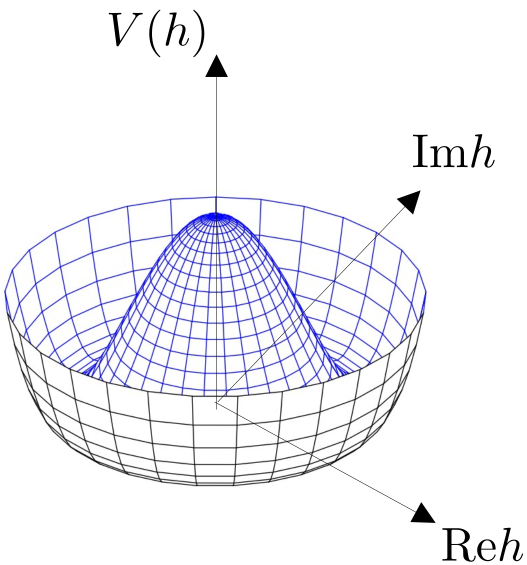

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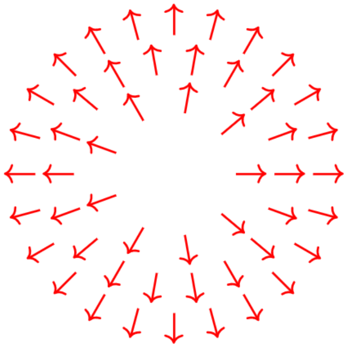

Winding number 1

Cosmic String

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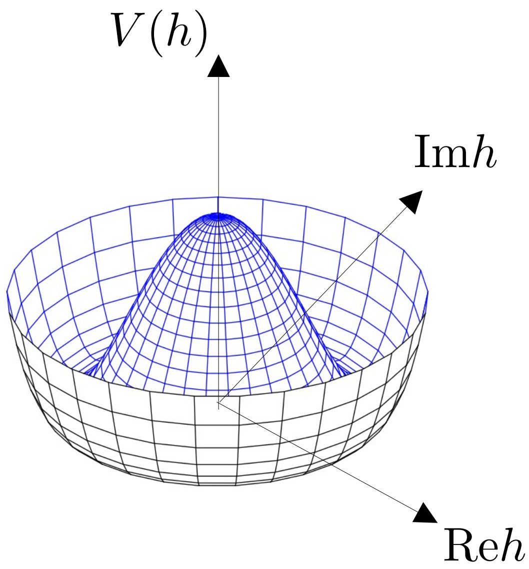

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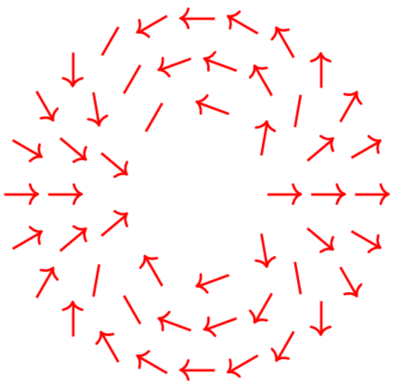

Winding number 2

Cosmic String

$$\mathrm{U}(1) \to \text{nothing}$$

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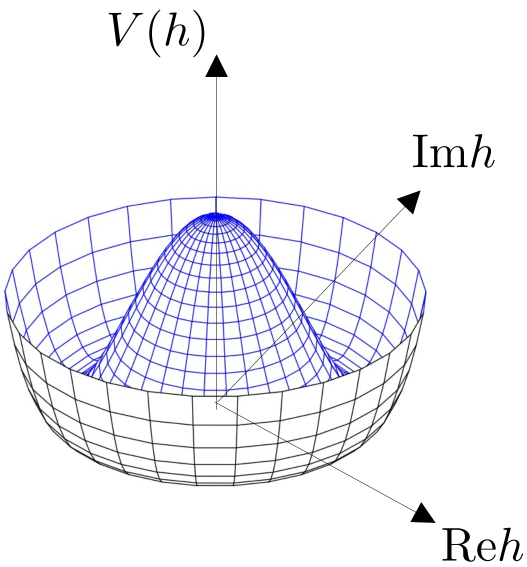

Spontaneous
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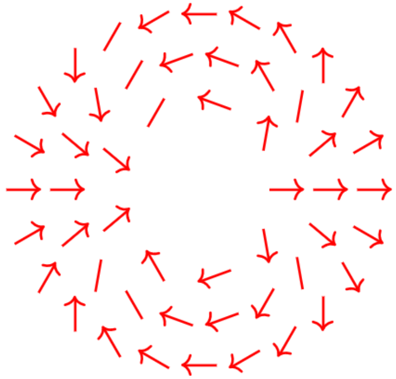
Phase of h

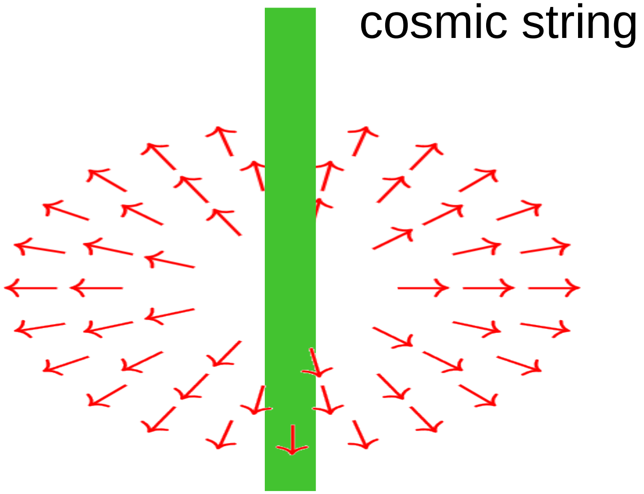


Winding number 2

$$\pi_1(\mathrm{U}(1)) = \mathbb{Z}$$

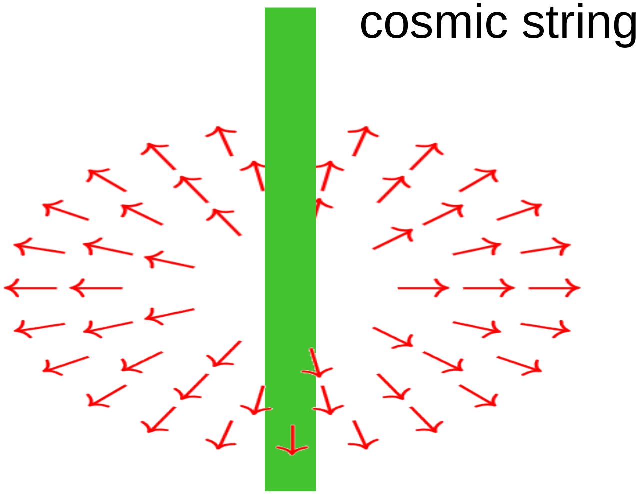
Cosmic String

$$\mathrm{U}(1) \to \text{nothing}$$



Cosmic String

$$\mathrm{U}(1)_{\mathrm{gauge}} \to \mathrm{nothing}$$



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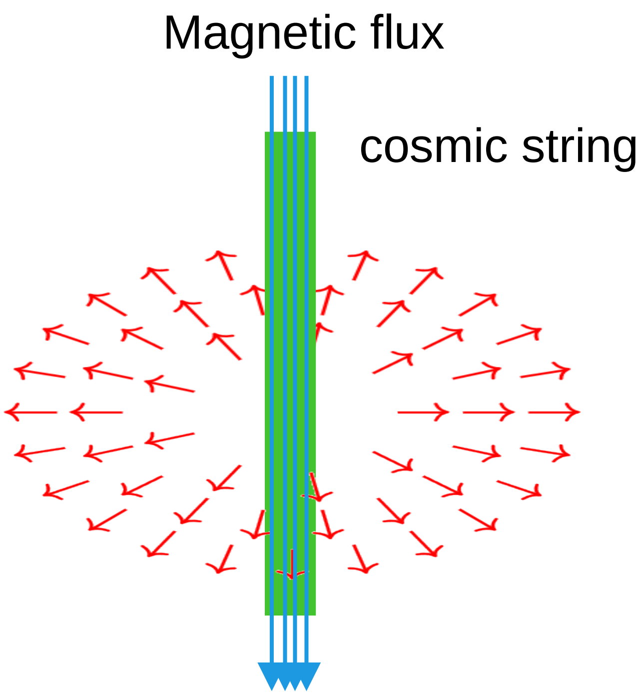

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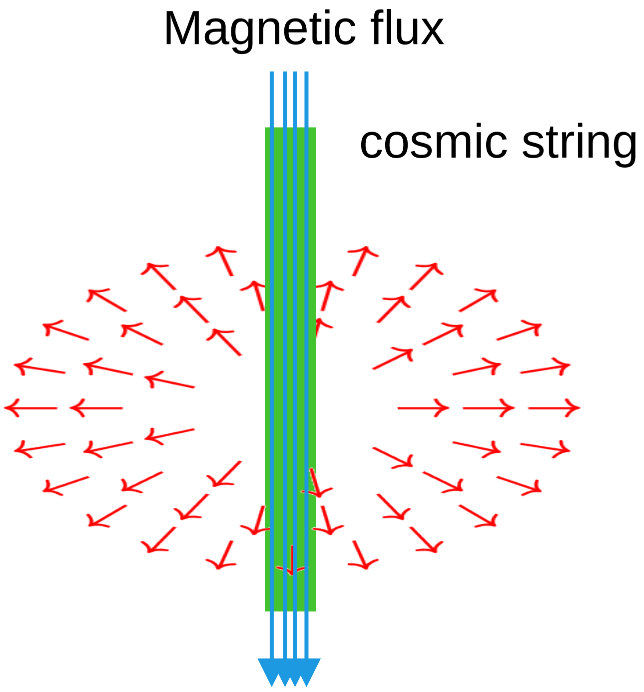

$$\nabla \cdot \vec{B} = 0$$

Multi-step SSB

$$\mathrm{SU}(2) \to \mathrm{nothing}$$

$$\pi_2(\mathrm{SU}(2)) = \pi_1(\mathrm{SU}(2)) = 0$$

No topological objects.

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Monopole Cosmic string

⬇

Metastable string with $V \gg v$

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Metastable string with $V \gg v$

Example:

$$\mathrm{SO}(10)_{\mathrm{GUT}} \to G_{\mathrm{SM}} \times \mathrm{U}(1)_{B-L} \to G_{\mathrm{SM}}$$

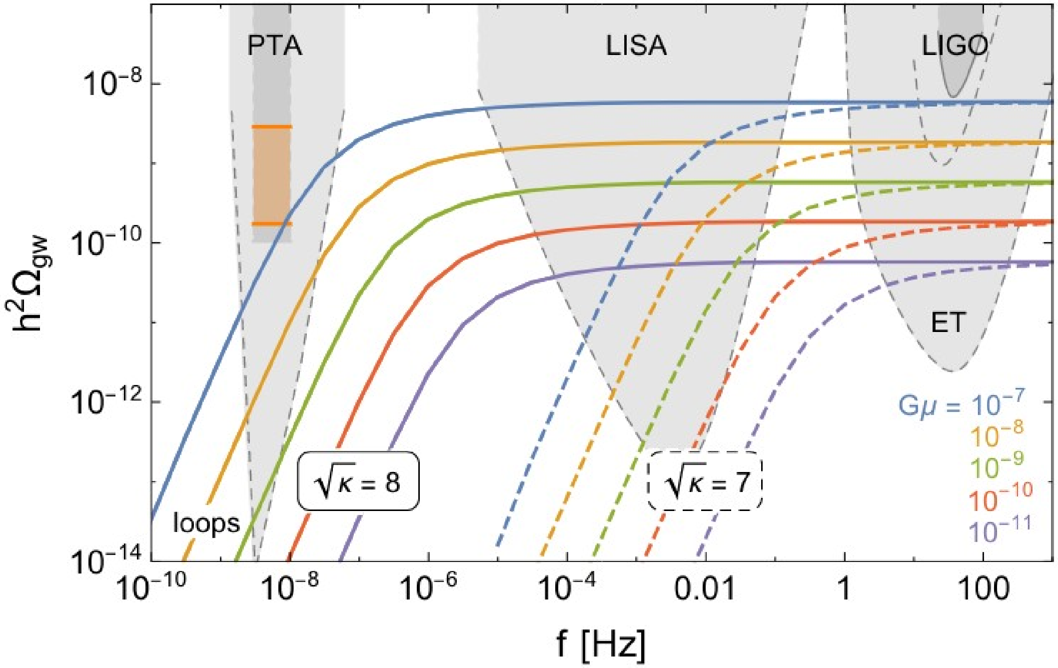
Gravitational Wave

String breaking rate per unit length

$$\Gamma = \frac{\mu}{2\pi} e^{-\pi\kappa}$$

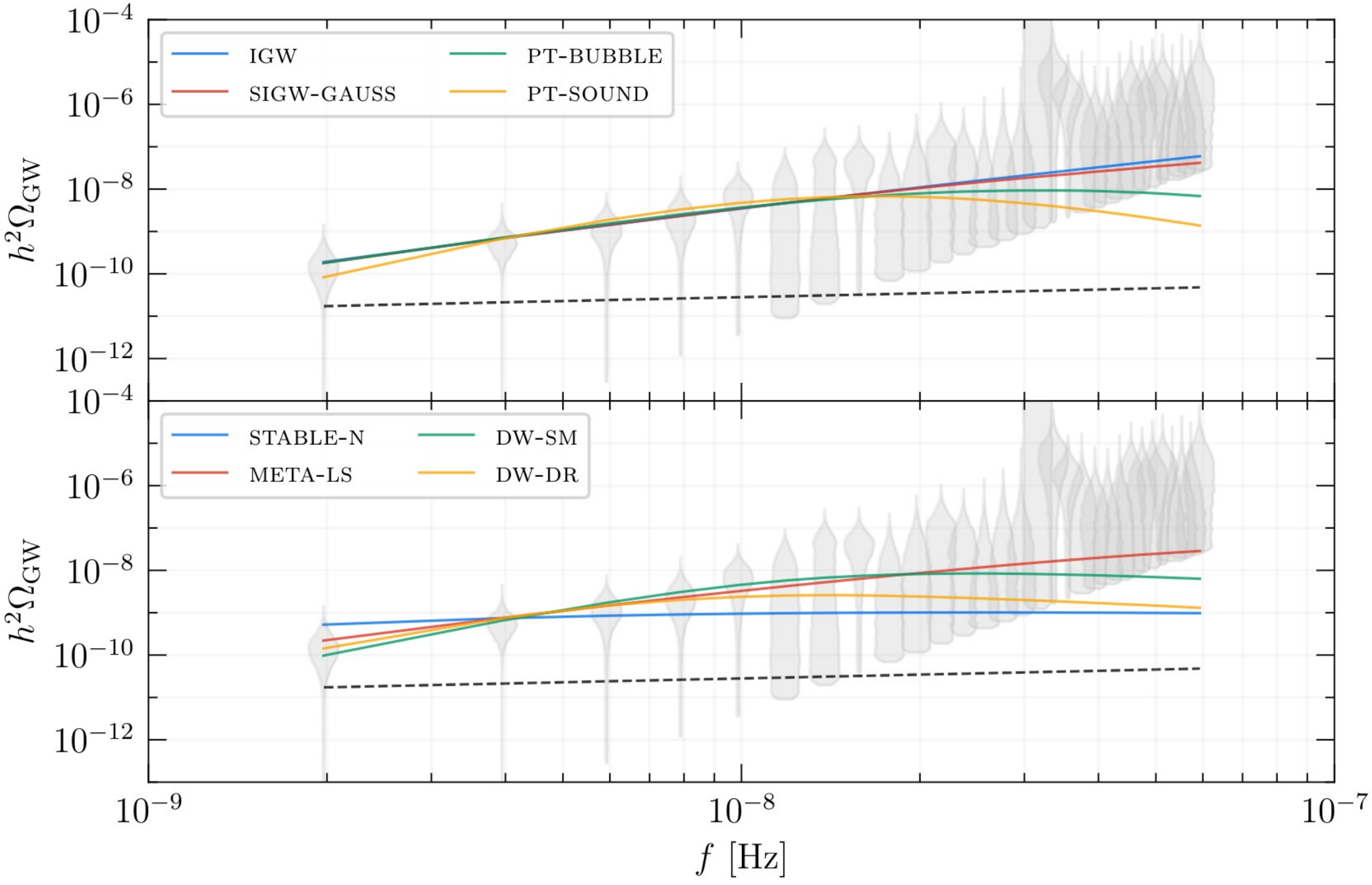
μ: String tension

[Buchmüller, Domcke, Schmitz 2107.04578]



Hint of Metastable String?

[NANOGrav, 2306.16219]



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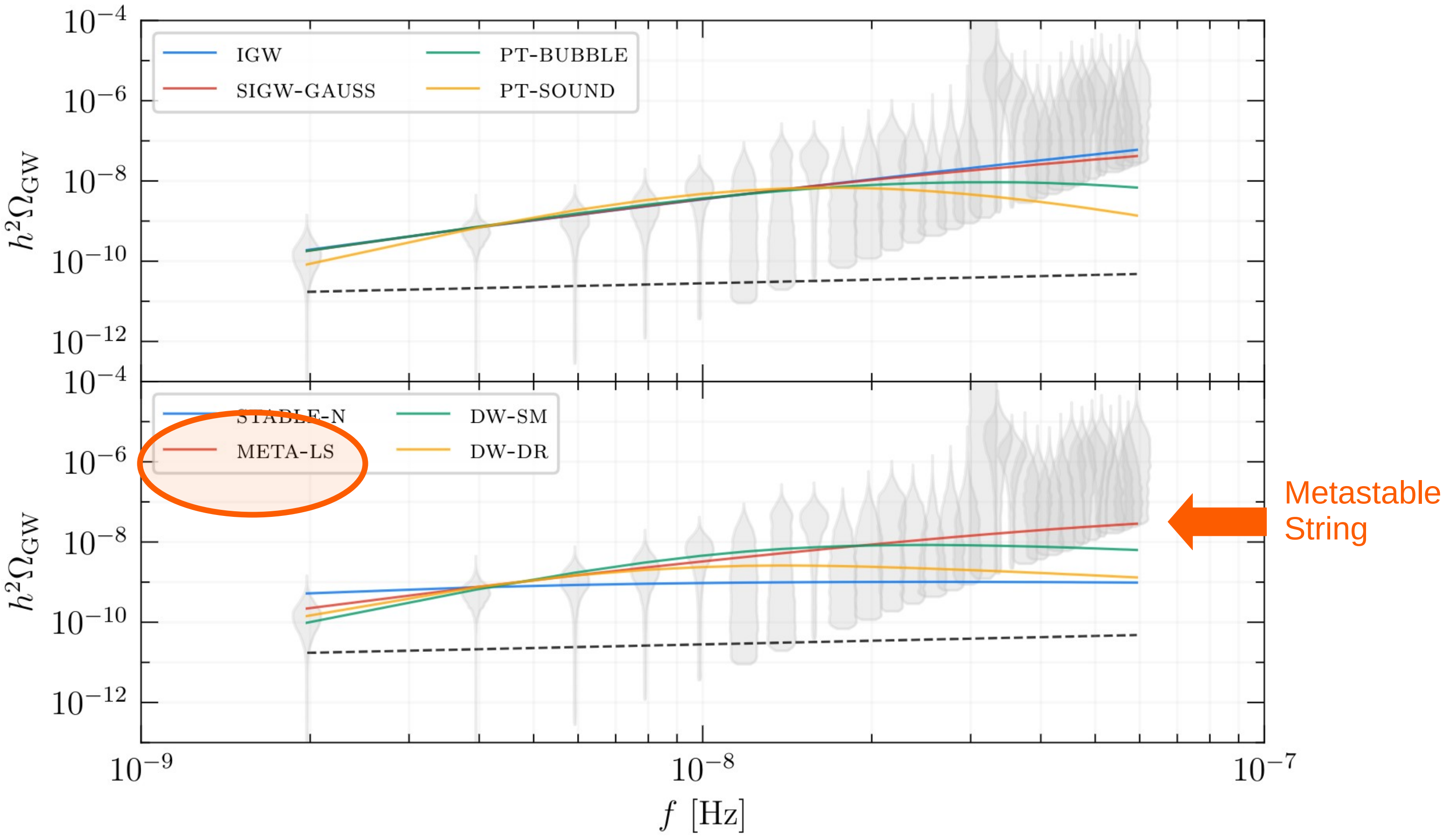

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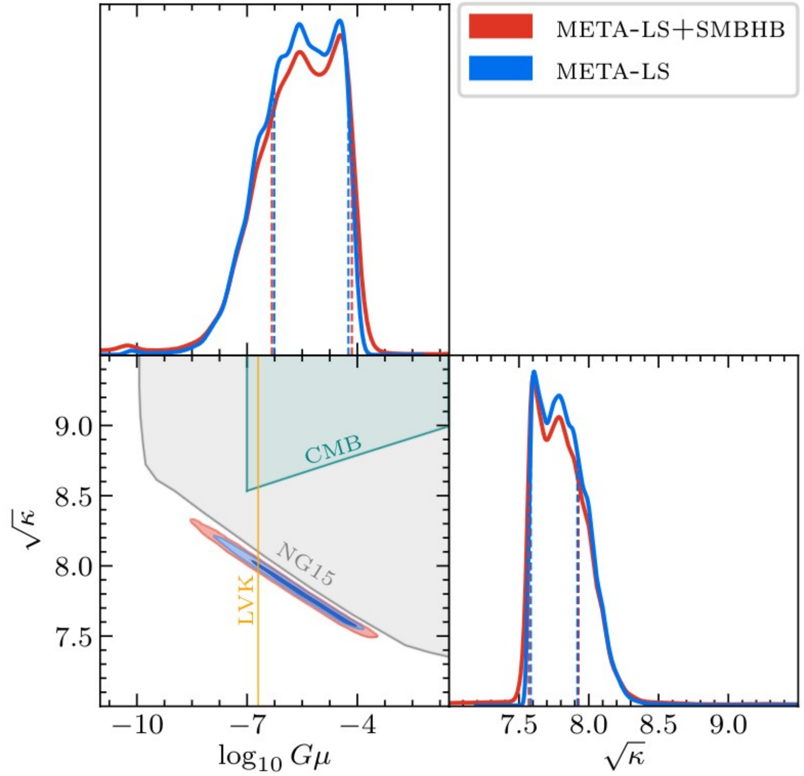

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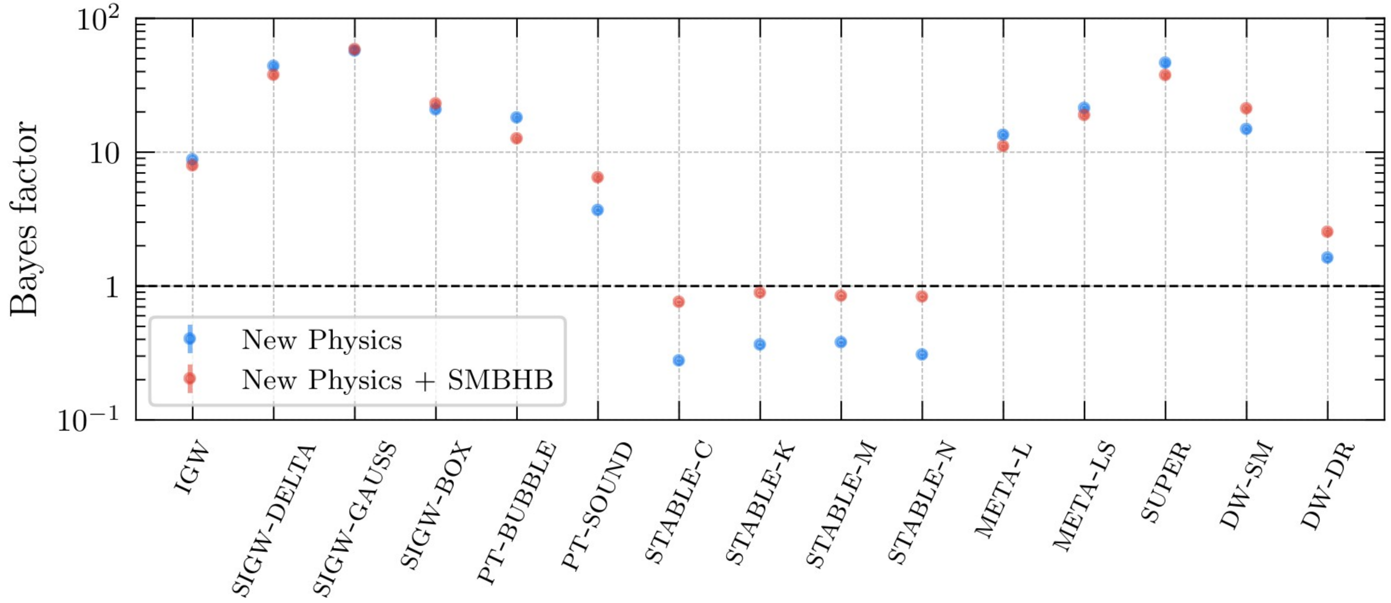

Bayes factor ~ Fitness of model for data.

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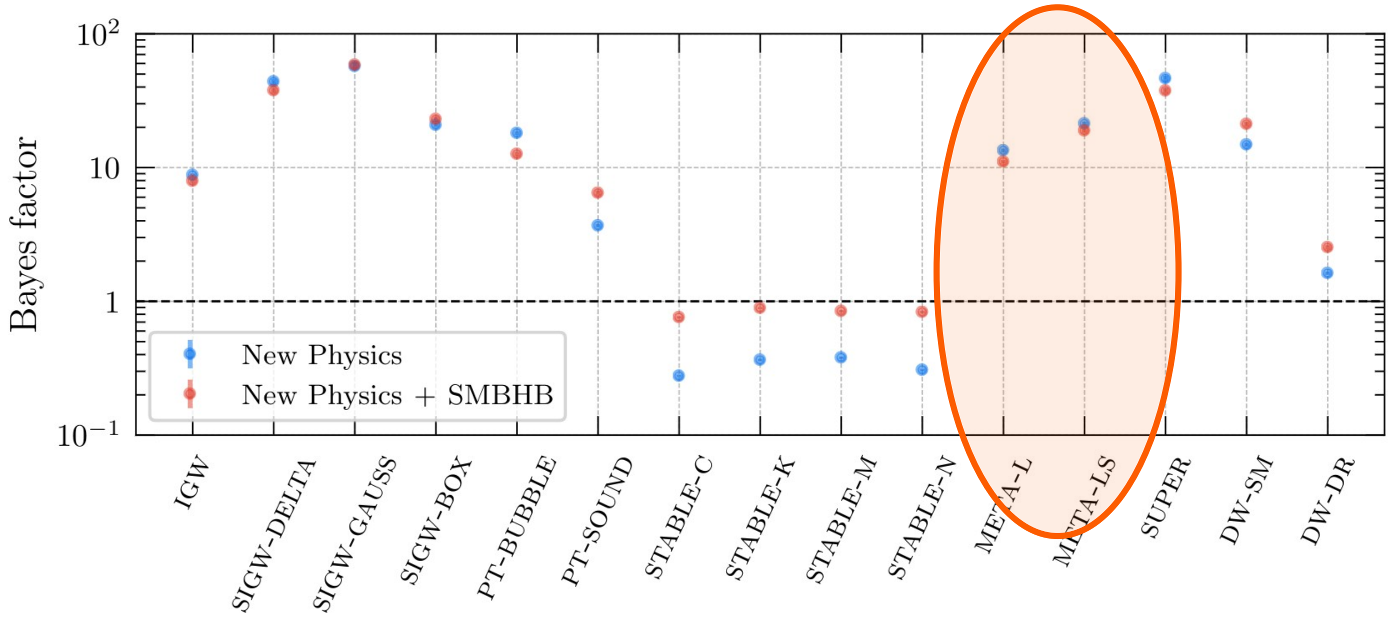
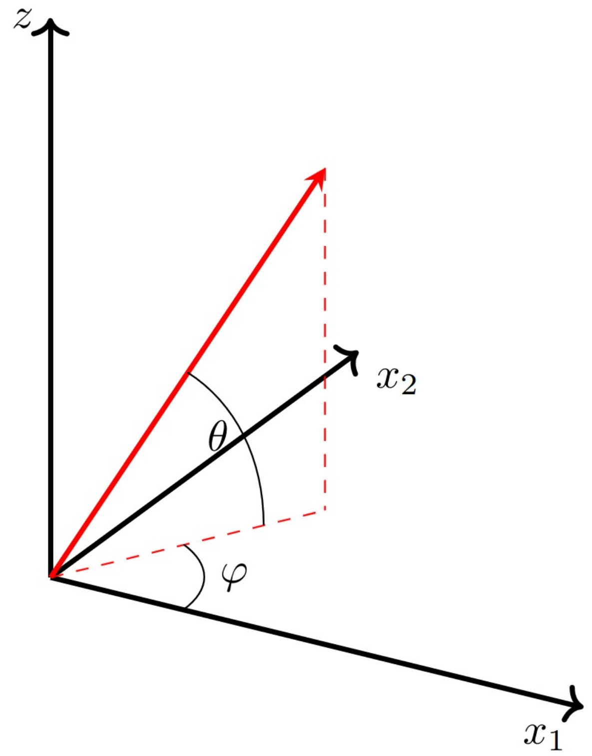


Bayes factor ~ Fitness of model for data.

Breaking of Metastable String with Monopole

Coordinate

String direction



Explicit Model: SU(2) Theory

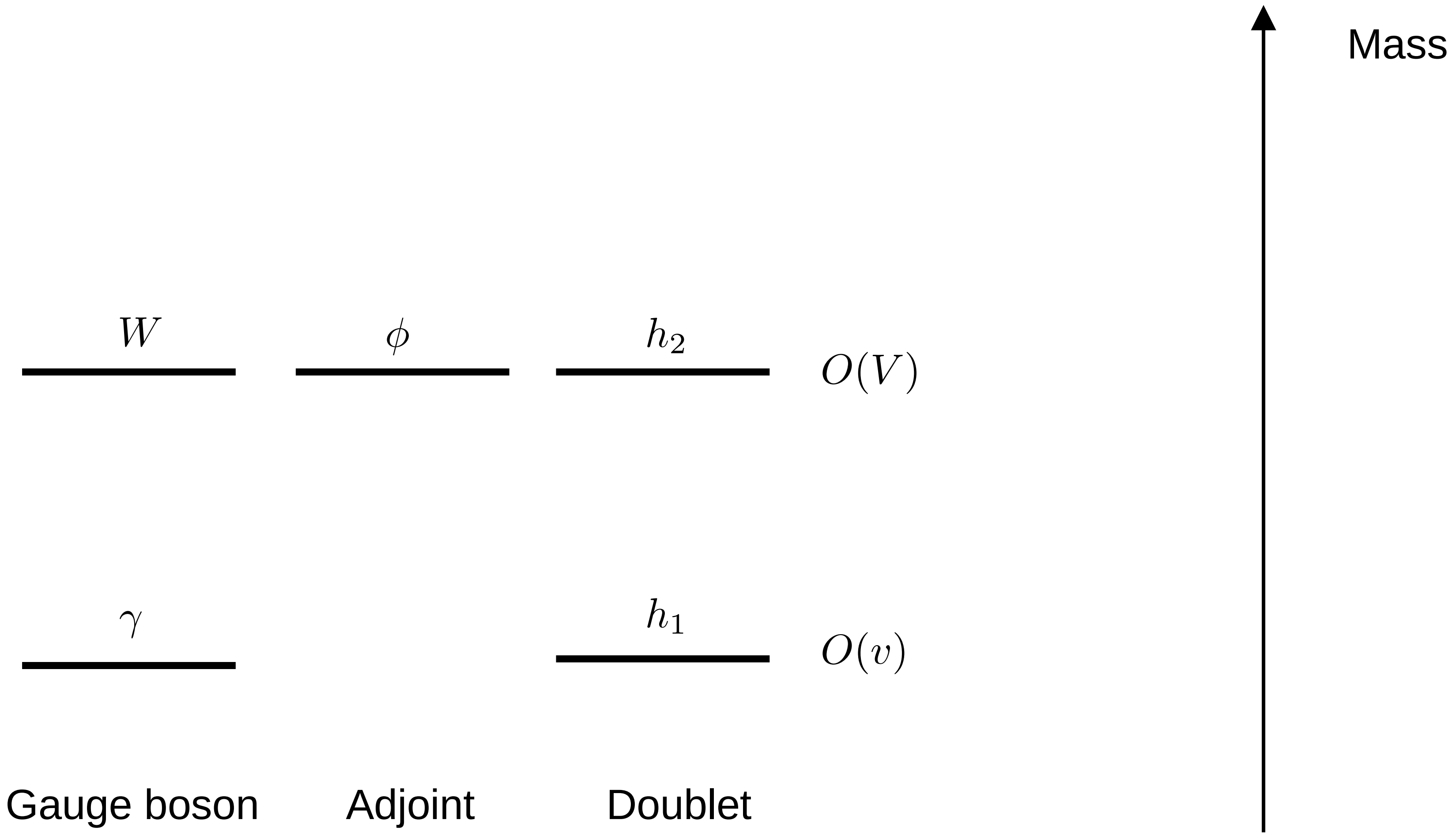
Symmetry breaking pattern:

$$\mathrm{SU}(2) \underset{V}{\to} \mathrm{U}(1) \underset{v}{\to} \text{ nothing}$$

Higgs fields:

ϕ	Triplet	$\langle \phi^a \rangle = V \delta^{a3}$
h	Doublet	$\langle h \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$

Mass Spectrum



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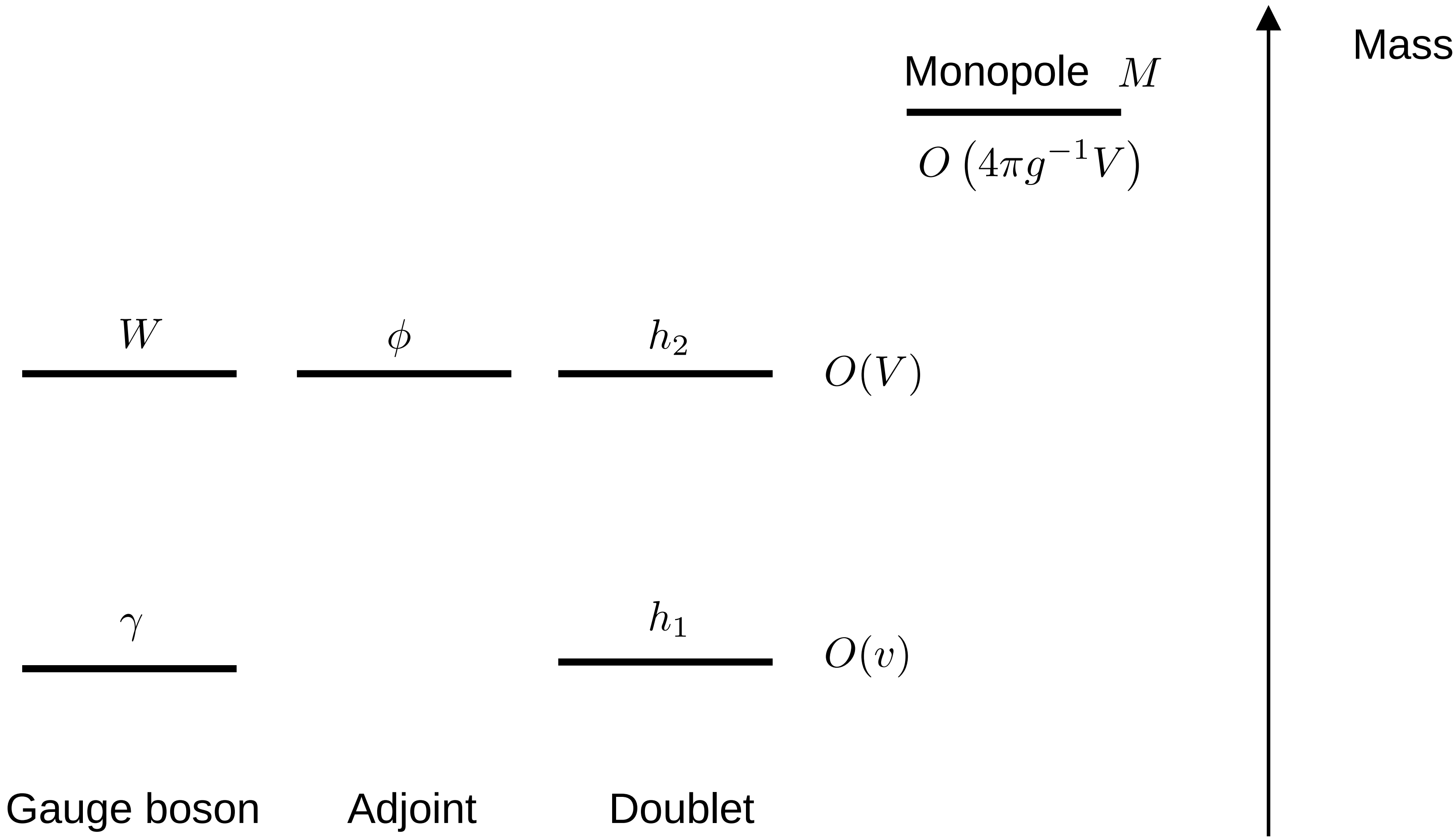
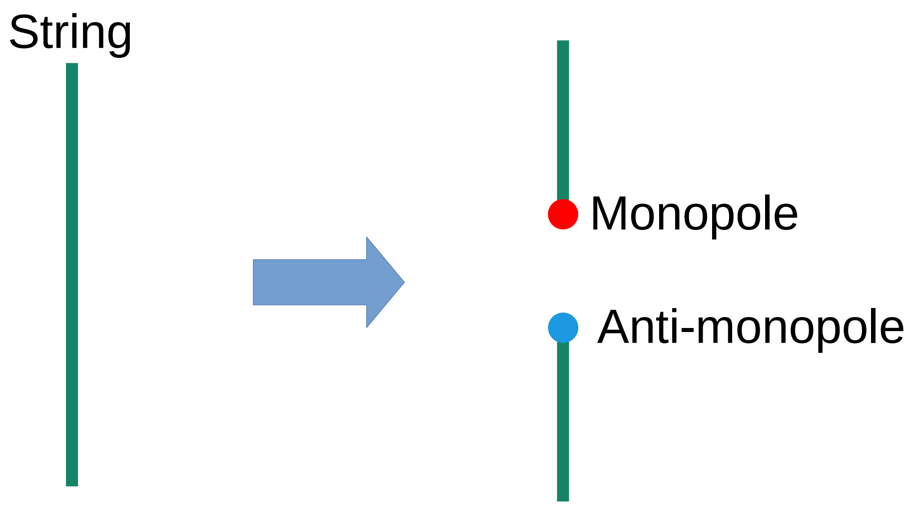


String Breaking via Monopole

[Vilenkin, NPB 196(1982) 240]
[Preskill & Vilenkin, hep-ph/9209210]



Monopole pair production like Schwinger effect.

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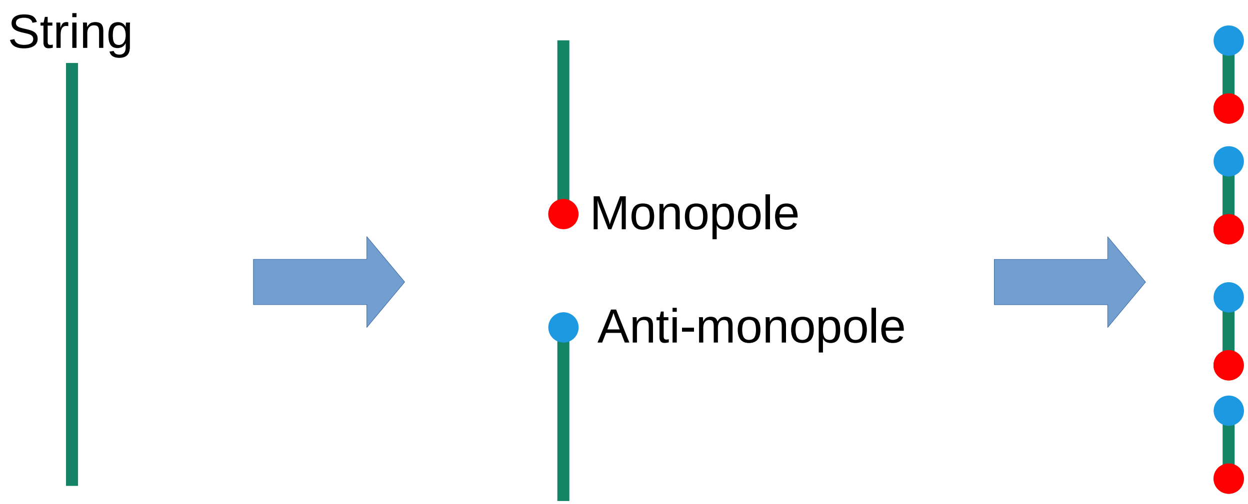
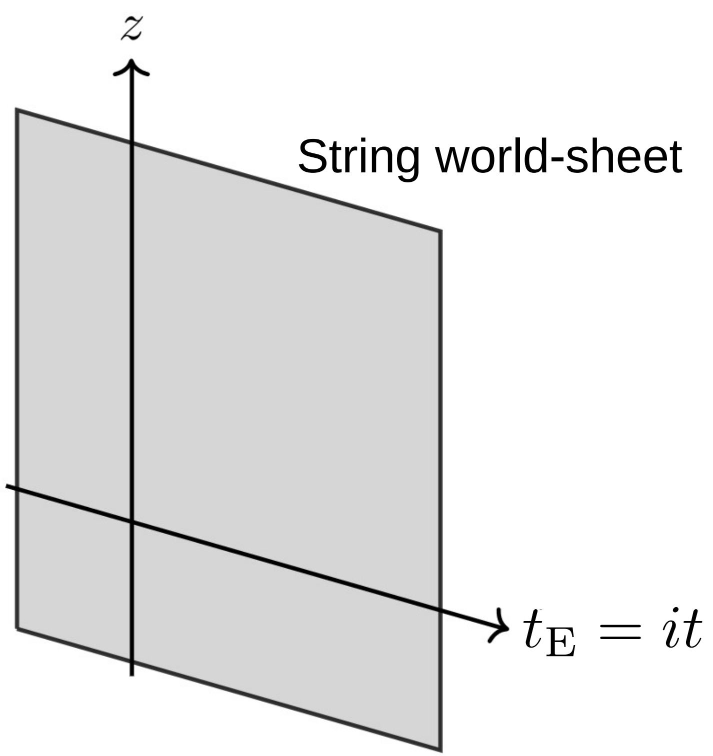


Monopole pair production like Schwinger effect.

Conventional Approach

$V \gg v$ Infinitely thin string



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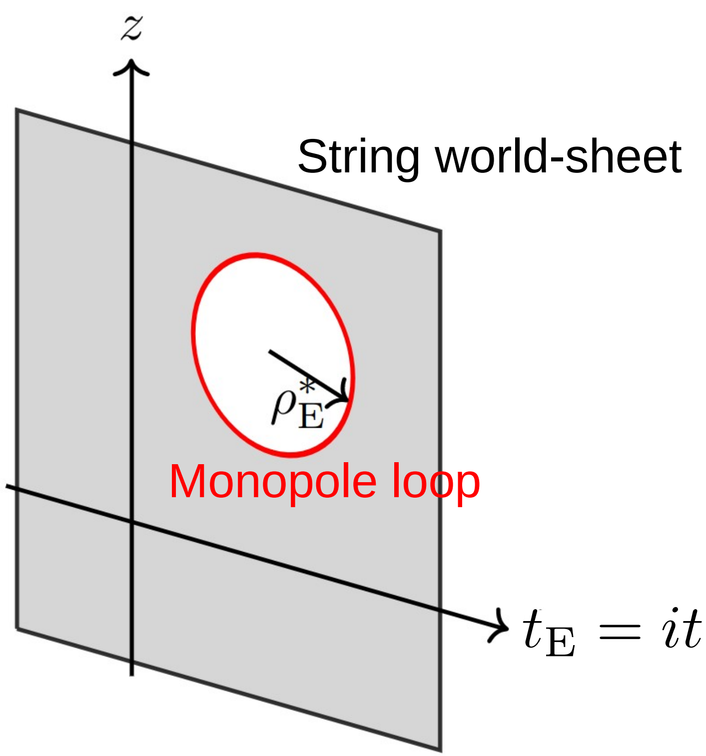

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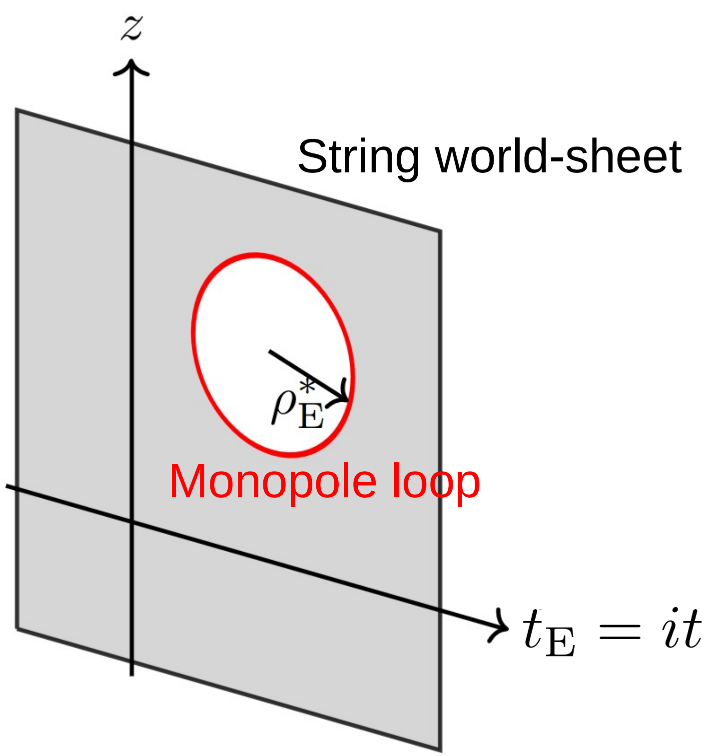

$$S_B = m_M \int_{\mathrm{worldline}} dx - \mu \int_{\mathrm{hole}} d^2 S$$

$$= 2\pi \rho_{\mathrm{E}}^* m_M - \pi {\rho_{\mathrm{E}}^*}^2 \mu$$

$$\Rightarrow \quad \rho_{\mathrm{E}}^* = \frac{m_M}{\mu}$$

maximization

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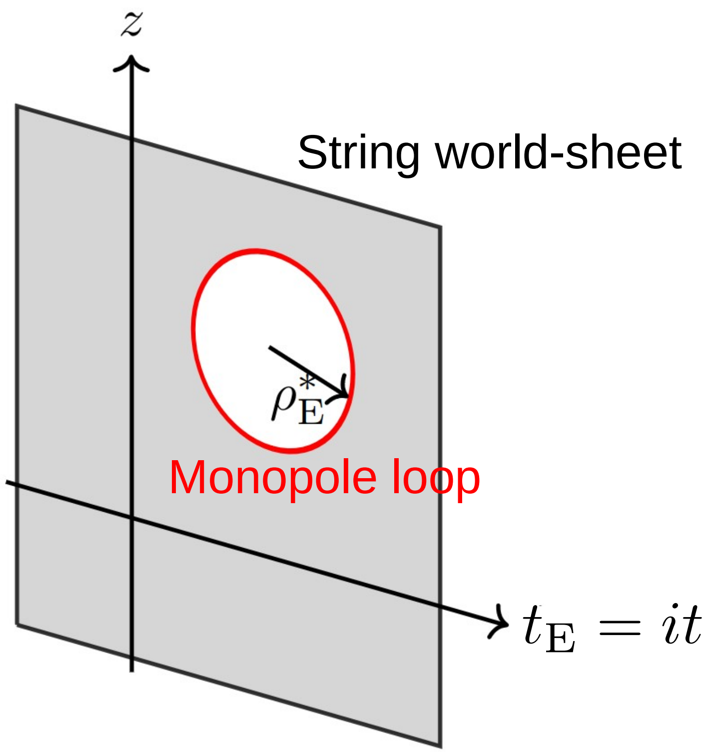

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$$\Rightarrow \quad \rho^*_{\mathrm{E}} = \frac{m_M}{\mu}$$

maximization

$$S_B^{(\text{thin})} = \frac{\pi m_M^2}{\mu} =: \pi\kappa$$

$$\Gamma \propto e^{-S_B^{(\text{thin})}}$$

Nanograv Implication

NANOGrav

$$\sqrt{\kappa} = \frac{m_M}{\sqrt{\mu}} = O\left(\frac{4\pi V}{\sqrt{2\pi} g v}\right) \simeq 8 \quad \Rightarrow \quad V/v = O(1)$$

$$\mu \sim 2\pi v^2$$

$$M_M \sim 4\pi g^{-1} V$$

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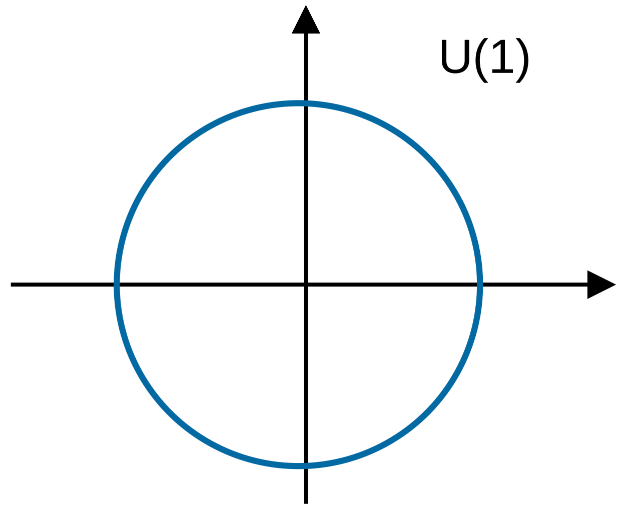
Any other path to break the string?

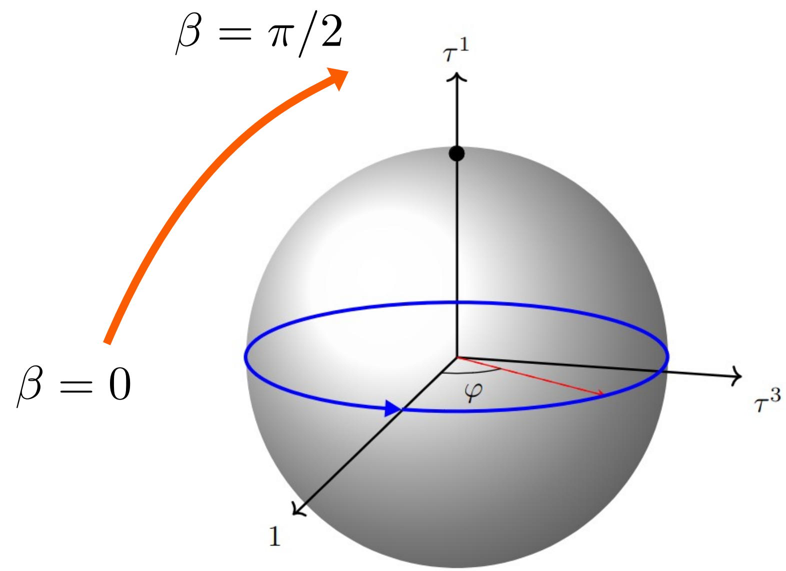
Explicitly construct bounce configuration of scalar and vector fields.

Revisiting String Breaking

U(1) in SU(2)



$\pi_1(\mathrm{U}(1)) \neq 0$

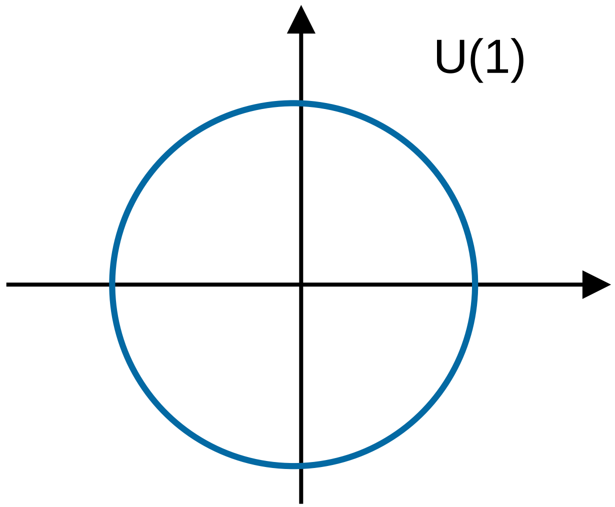


$\pi_1(\mathrm{SU}(2)) = 0$

β : Unwinding parameter.

β → π/2, no winding.

U(1) in SU(2)



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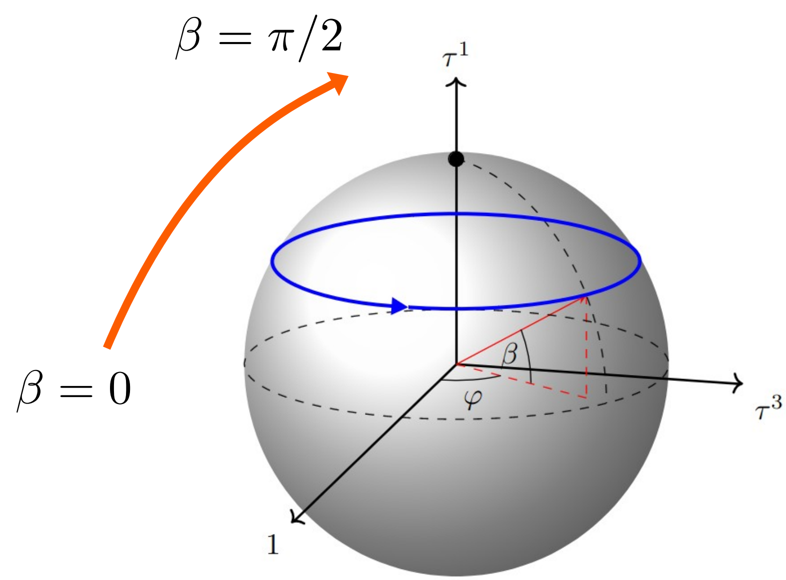

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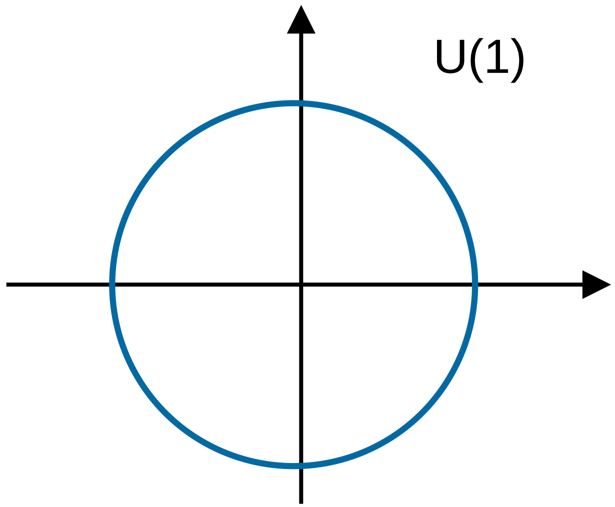

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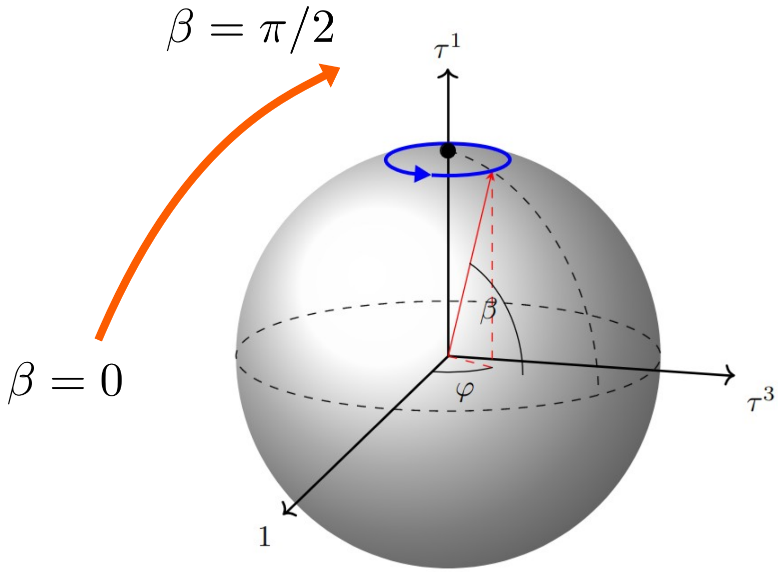

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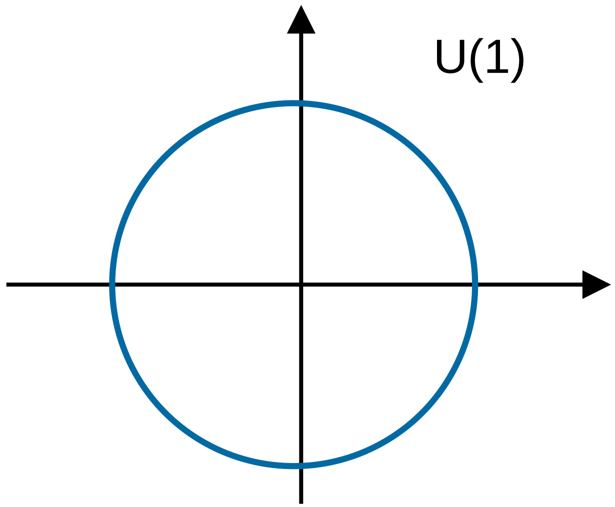

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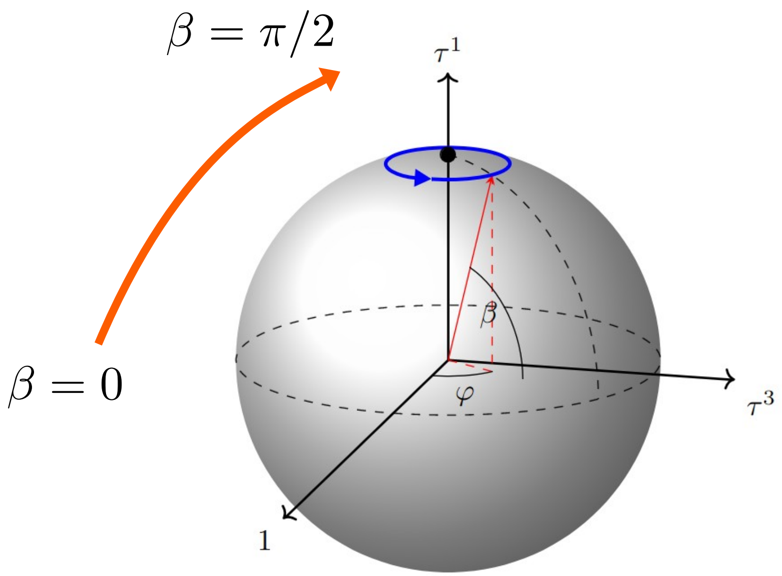

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β : Unwinding parameter.

β → π/2, no winding.

Using β, explicit field configuration of string breaking is possible.

Primitive Ansatz

[Shifman & Yung hep-th/0205025]

τ : Pauli matrices

$\rho = \sqrt{x_1^2 + x_2^2}$

$$\boxed{U = e^{-i\varphi\tau^3}\cos\beta + i\tau^1 \sin\beta}$$

$$h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$$

$$A_\rho(x) = 0$$

$$A_\varphi(x) = iU(\partial_\varphi U^\dagger)(1 - f_\beta(\rho))$$

$$\phi(x) = VU\frac{\tau^3}{2}U^\dagger + \Delta\phi$$

$$\Delta\phi = \Phi_\beta(\rho)\left(\frac{\tau^1}{2}\sin\varphi - \frac{\tau^2}{2}\cos\varphi\right)$$

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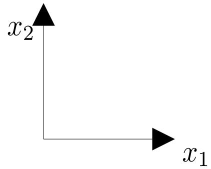
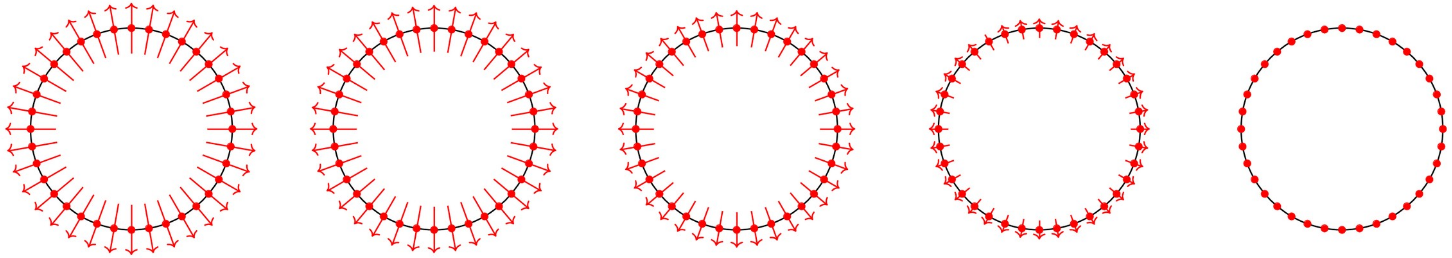
These fields yet to be determined.

Field Configuration

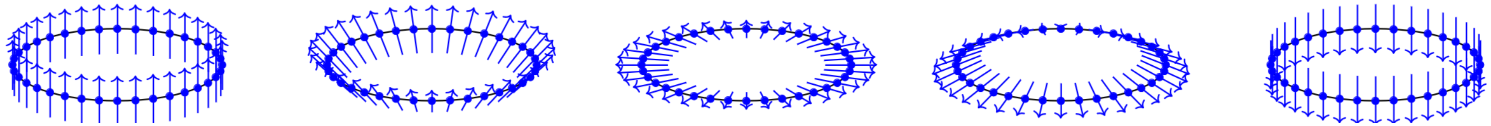


φ dependent phase of doublet h



$\beta = 0$ $\beta = \pi/8$ $\beta = \pi/4$ $\beta = 3\pi/4$ $\beta = \pi/2$

Direction of Adjoint field Φ^a



Procedure

- Promote β as collective coordinate β(t,z).

- Get effective action.

$$S_{\mathrm{Eucl}} = 2\pi \int \rho_{\mathrm{E}} d\rho_{\mathrm{E}} \left(\frac{1}{2} K_{\mathrm{eff}}(\beta)(\partial_{\rho_{\mathrm{E}}} \beta(\rho_{\mathrm{E}}))^2 + T(\beta) \right)$$

$$\rho_{\mathrm{E}} = \sqrt{t_{\mathrm{E}}^2 + z^2}$$

- Get tunneling rate with bounce solution.

Primitive Ansatz

$$h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$$

$$A_\rho(x) = 0$$

$$A_\varphi(x) = iU(\partial_\varphi U^\dagger)(1 - f_\beta(\rho))$$

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$$\Delta\phi = \Phi_\beta(\rho)\left(\frac{\tau^1}{2}\sin\varphi - \frac{\tau^2}{2}\cos\varphi\right)$$

$$A_{t/z}(x) = -2(\partial_{t/z}\beta(t,z))\left(\frac{\tau^1}{2}\cos\varphi - \frac{\tau^2}{2}\sin\varphi\right)a_\beta(\rho)$$

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Substitute to original Lagrangian

(kinetic-term) + V_{Higgs}

$$(\partial_{\rho_{\mathrm{E}}}\beta)^2 K_{\mathrm{eff}}[\xi_\beta, f_\beta, \Phi_\beta, a_\beta] + T[\xi_\beta, f_\beta, \Phi_\beta] \qquad \rho_{\mathrm{E}} = \sqrt{t_{\mathrm{E}}^2 + z^2}$$

String Tension

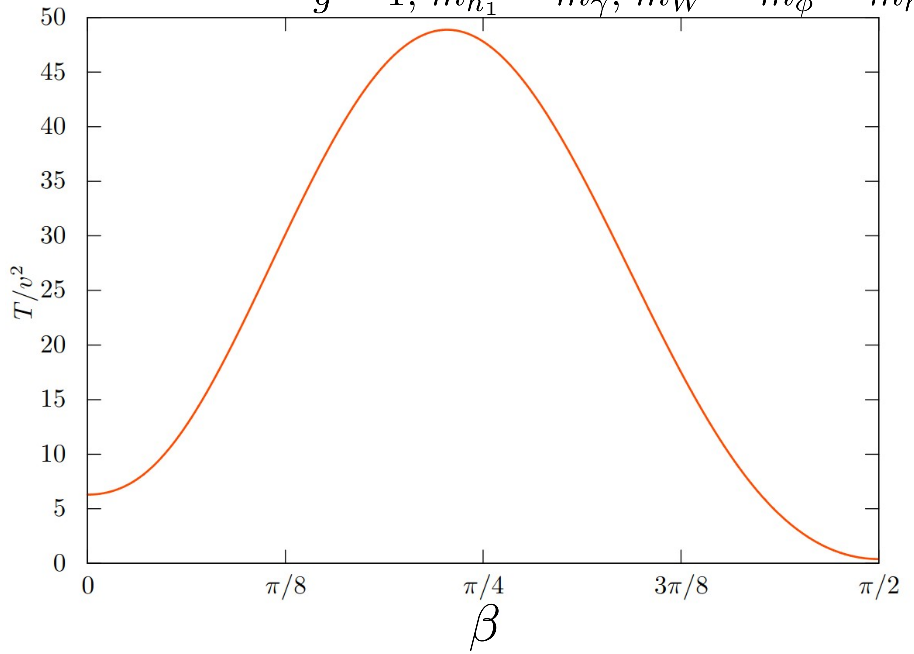
Determine f, Φ, ξ to minimize string tension for each β.

$$\rho = \sqrt{x_1^2 + x_2^2}$$

$$T(\beta) = 2\pi \int_0^\infty \rho\, \mathrm{d}\rho \left\{ \frac{2}{g^2} \frac{(\partial_\rho f_\beta)^2}{\rho^2} \cos^2\beta + (\partial_\rho \xi_\beta)^2 + \frac{f_\beta^2}{\rho^2} \xi_\beta^2 \cos^2\beta \right.$$
$$+ \frac{1}{2}(\partial_\rho \Phi_\beta)^2 + \frac{1}{2\rho^2}\left[\Phi_\beta(\cos 2\beta - 2 f_\beta \cos^2\beta) + V f_\beta \sin 2\beta\right]^2$$
$$\left. + \lambda(\xi_\beta^2 - v^2) + \tilde{\lambda}\left[\Phi_\beta^2 - 2V\Phi_\beta \sin 2\beta\right]^2 + \frac{\gamma}{4}\Phi_\beta^2 \xi_\beta^2 \right\} .$$

String tension

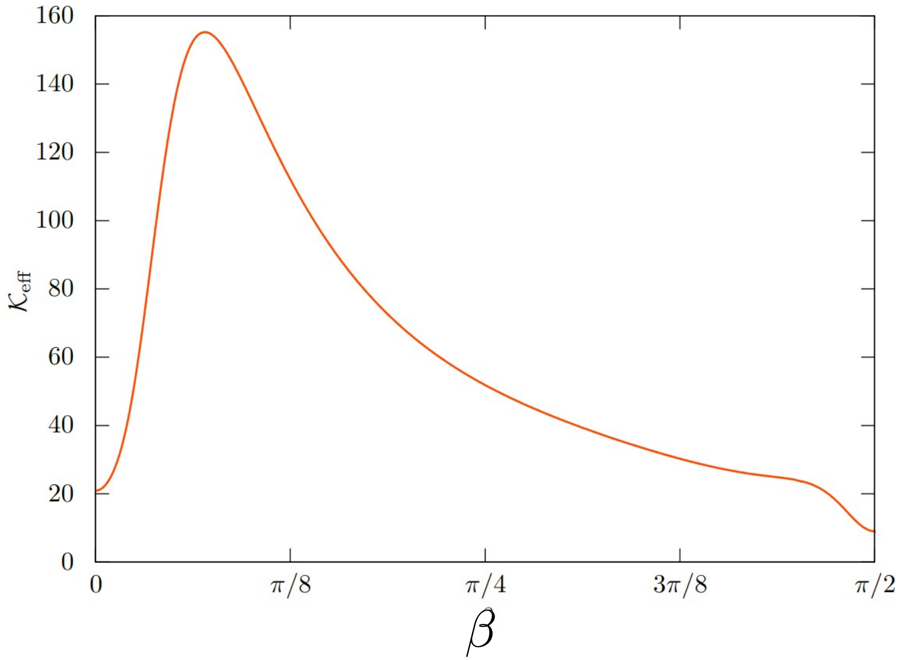
$g = 1,\ m_{h_1} = m_\gamma,\ m_W = m_\phi = m_{h_2} = 5 m_\gamma$



Kinetic Term

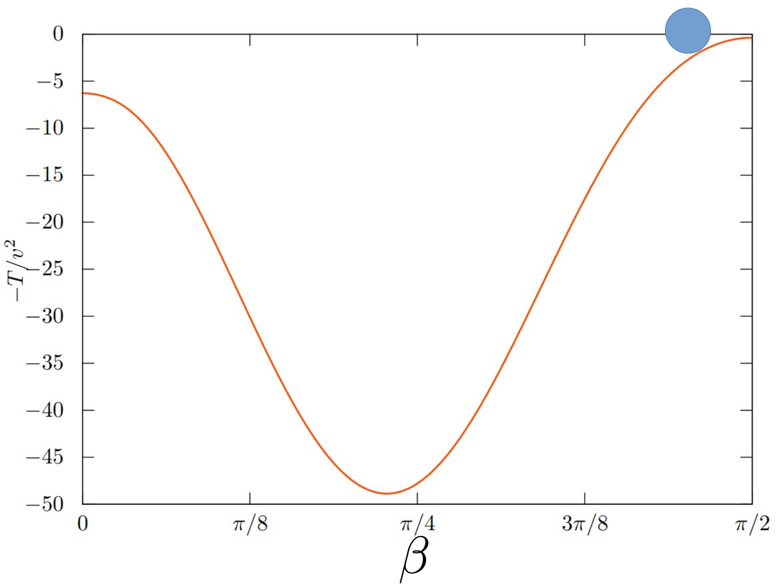
Determine a_β to minimize kinetic term for each β.

$$8\pi\rho\Bigg[\frac{1}{g^2}(\partial_\rho a_\beta)^2 \ -a_\beta\Big\{-2\Phi_\beta(V\sin 2\beta-\Phi_\beta)+\frac{1}{2}V(\partial_\beta\Phi_\beta)\cos 2\beta + \frac{1}{g^2\rho^2}\Big(f_\beta^2-\frac{1}{2}(\partial_\beta f_\beta)\sin 2\beta - f_\beta(1-f_\beta)\cos 2\beta\Big)\Big\} + a_\beta^2\Big\{\frac{1}{g^2\rho^2}(1-4f_\beta(1-f_\beta)\cos^2\beta)+V^2+\frac{1}{2}\xi_\beta^2+\Phi_\beta(-2V\sin 2\beta+\Phi_\beta)\Big\}\Bigg] \cdots$$



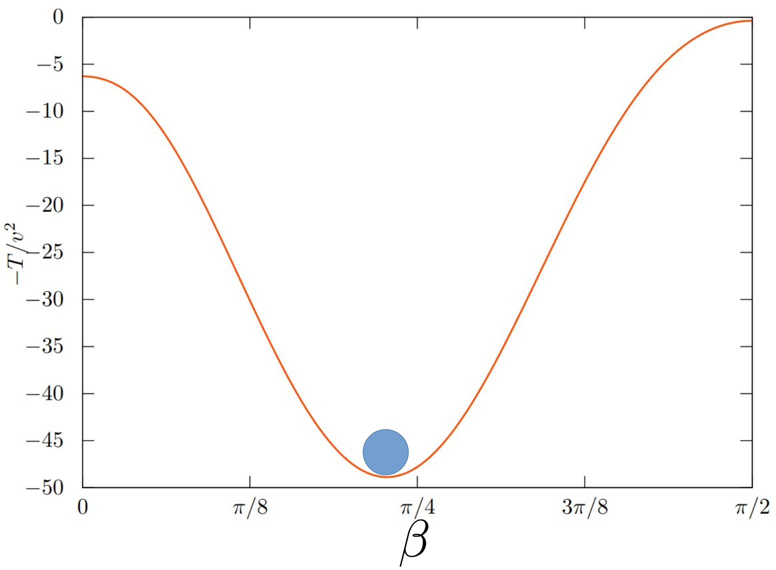
Effective Theory of β(ρ_E)

$$S_{\mathrm{Eucl}} = 2\pi \int \rho_{\mathrm{E}}\,\mathrm{d}\rho_{\mathrm{E}} \left[\frac{1}{2}\mathcal{K}_{\mathrm{eff}}(\beta(\rho_{\mathrm{E}}))(\partial_{\rho_{\mathrm{E}}}\beta(\rho_{\mathrm{E}}))^2 + T(\beta(\rho_{\mathrm{E}}))\right]$$



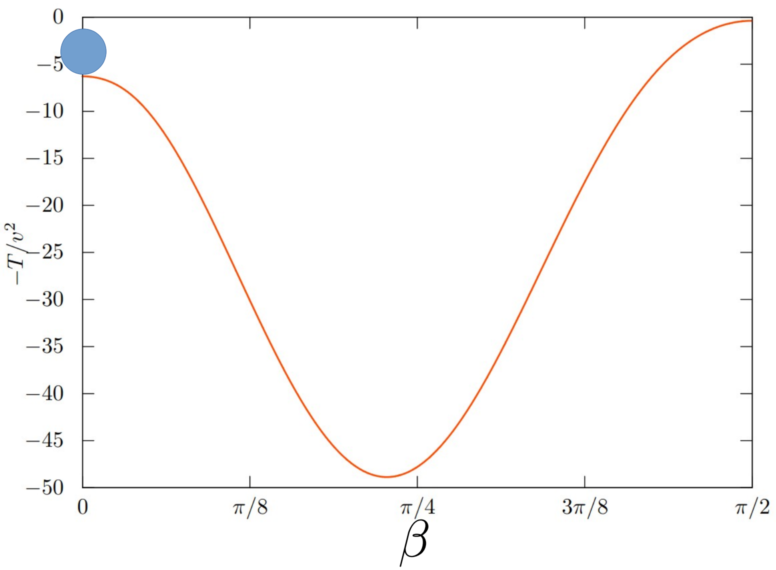
Effective Theory of β(ρ_E)

$$S_{\mathrm{Eucl}} = 2\pi \int \rho_{\mathrm{E}}\, \mathrm{d}\rho_{\mathrm{E}} \left[\frac{1}{2} \mathcal{K}_{\mathrm{eff}}(\beta(\rho_{\mathrm{E}}))(\partial_{\rho_{\mathrm{E}}} \beta(\rho_{\mathrm{E}}))^2 + T(\beta(\rho_{\mathrm{E}})) \right]$$



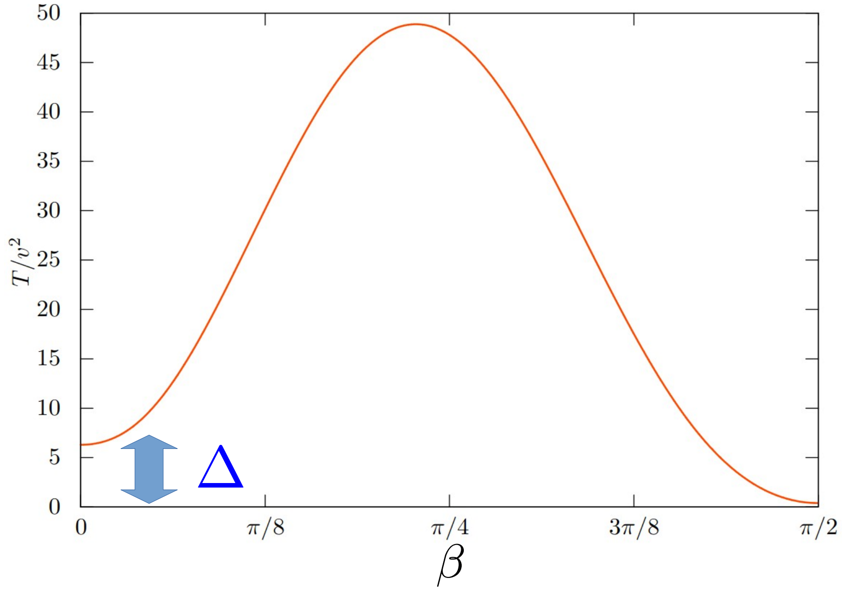
Effective Theory of β(ρ$_E$)

$$S_{\mathrm{Eucl}} = 2\pi \int \rho_{\mathrm{E}}\,\mathrm{d}\rho_{\mathrm{E}} \left[\frac{1}{2}\mathcal{K}_{\mathrm{eff}}(\beta(\rho_{\mathrm{E}}))(\partial_{\rho_{\mathrm{E}}}\beta(\rho_{\mathrm{E}}))^2 + T(\beta(\rho_{\mathrm{E}}))\right]$$



Thin-Wall Approximation

String tension $T(\beta)$



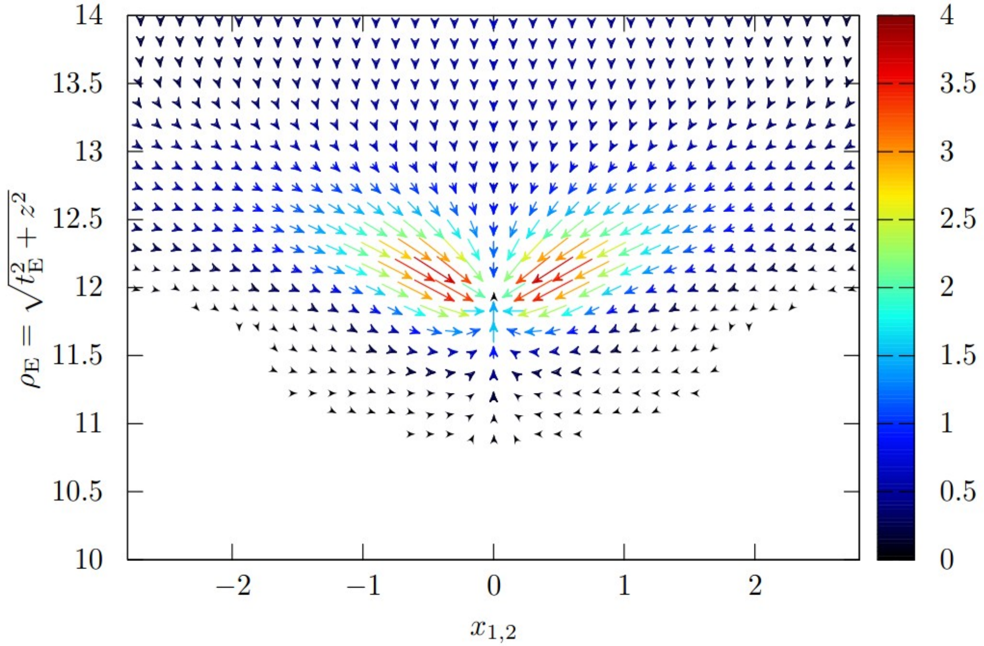
$\Delta \to 0$, thin-wall approximation is valid.

$$S_B \sim \frac{1}{\Delta}\left[\int_0^{2\pi}\sqrt{\mathcal{K}(\beta)(T(\beta)-\Delta)}d\beta\right]^2$$

Configuration during Tunneling

β = 0: String state

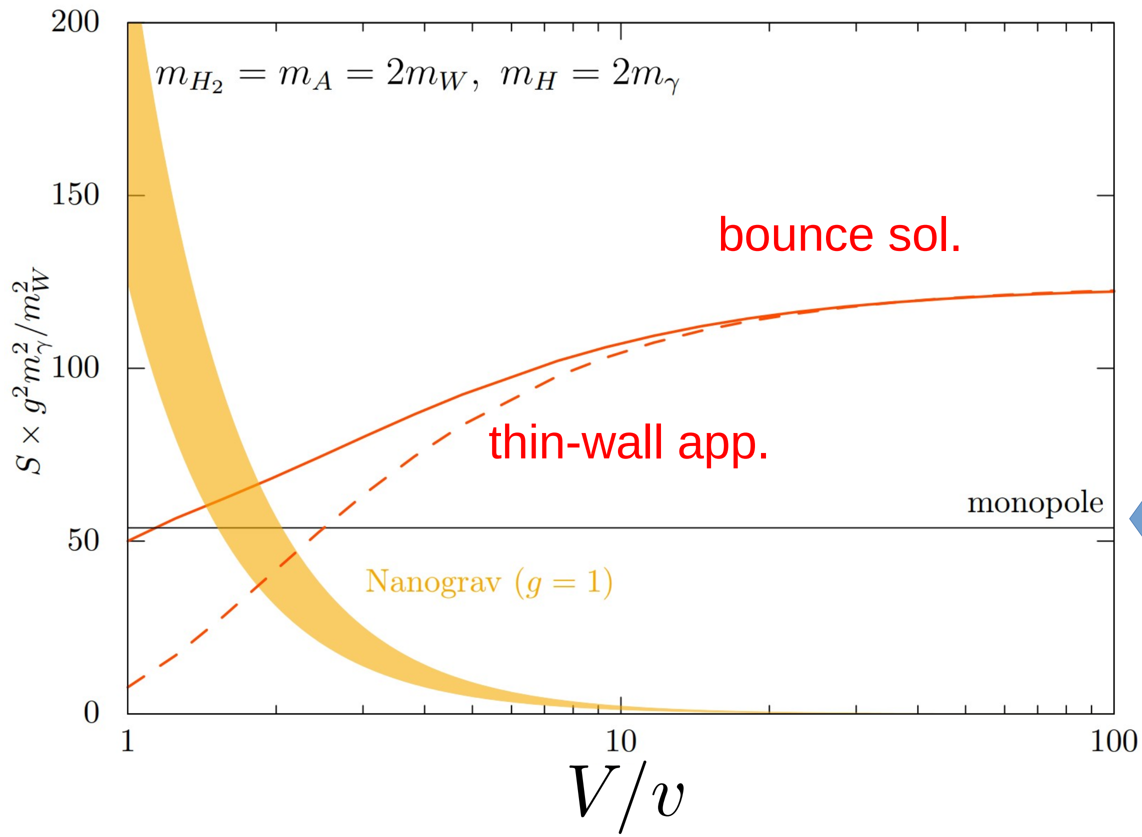
Magnetic field



β = π/2: “True” vacua

Bounce Action

Normalized S_B

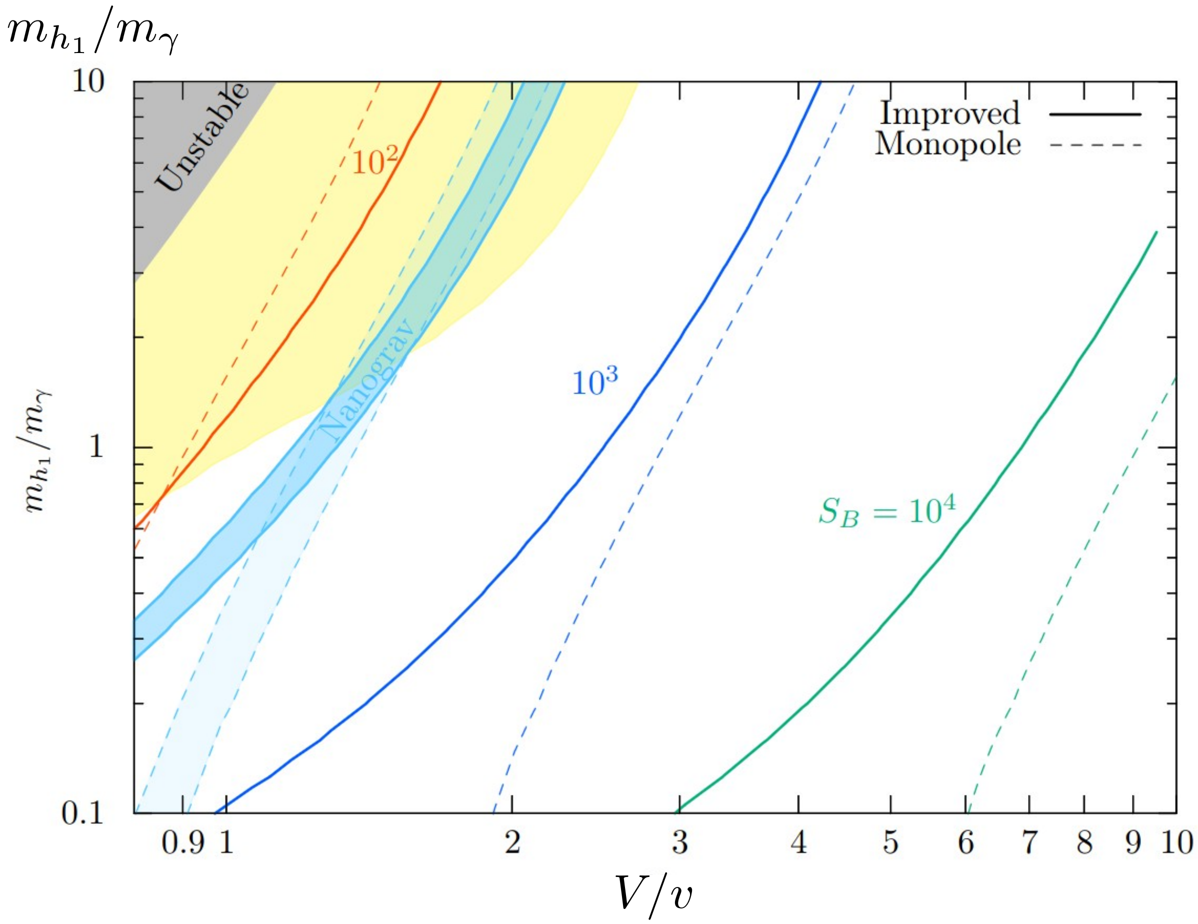


Conventional estimation

$$\frac{\pi m_M^2}{\mu}$$

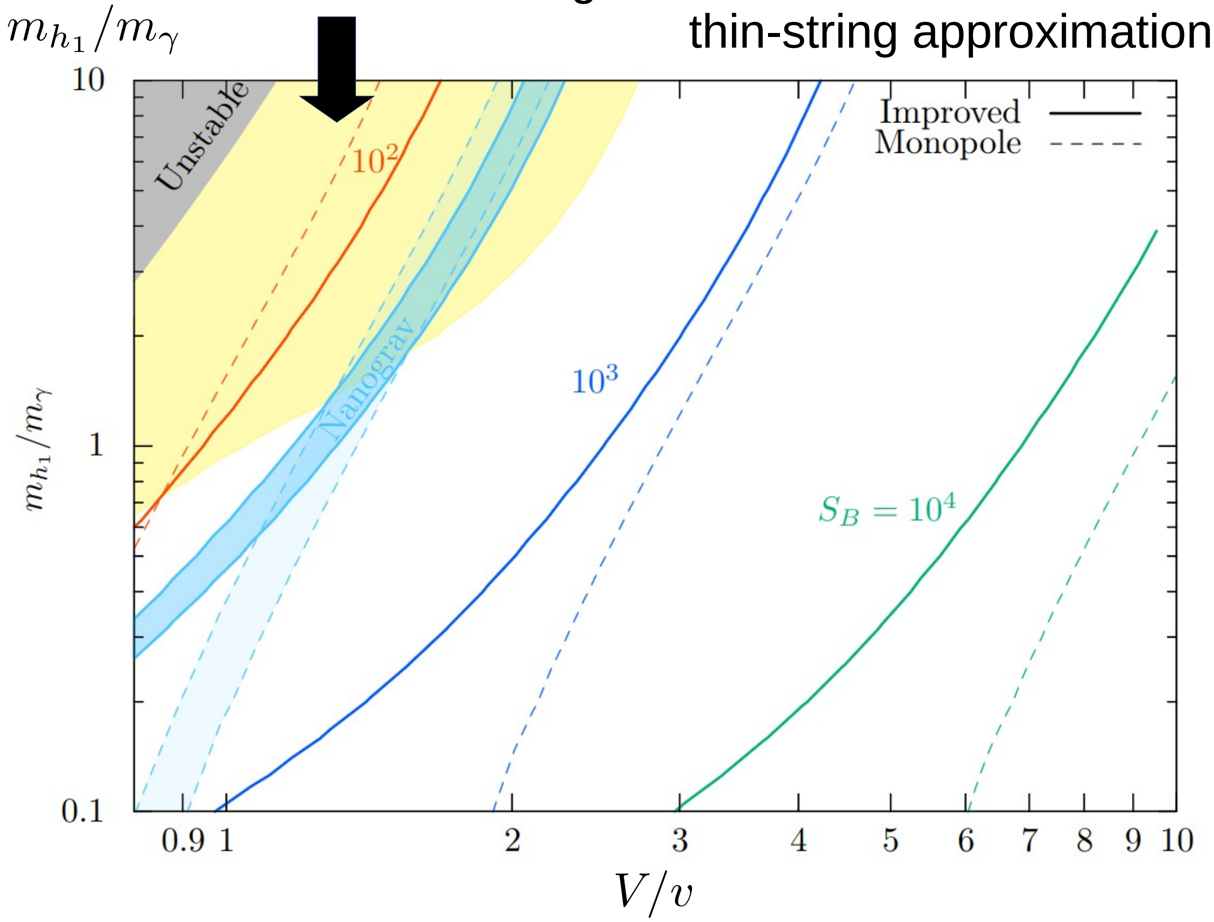
- The bounce solution results in larger SB.
- Thin-wall approximation fails when $V/v \lesssim 10.$

Higgs Mass Effect



Higgs Mass Effect

Yellow region: Bounce solution is “better” than thin-string approximation.



Summary and Prospect

- Metastable string can appear from hierarchical symmetry breaking.
  - GW from metastable string gives good fit to NANOGrav data.

- Breaking rate of metastable string is estimated.
  - Give robust lower limit of breaking rate even with $V/v = O(1)$.
  - Validity of conventional estimation?

- The present procedure is not best way to fastest path to string breaking.
  - Complete analysis without effective β description.

- Application to other metastable strings.

Explicit Model: SU(2) Theory

Symmetry breaking pattern:

$$\mathrm{SU}(2) \underset{V}{\rightarrow} \mathrm{U}(1) \underset{v}{\rightarrow} \text{ nothing}$$

Higgs fields:

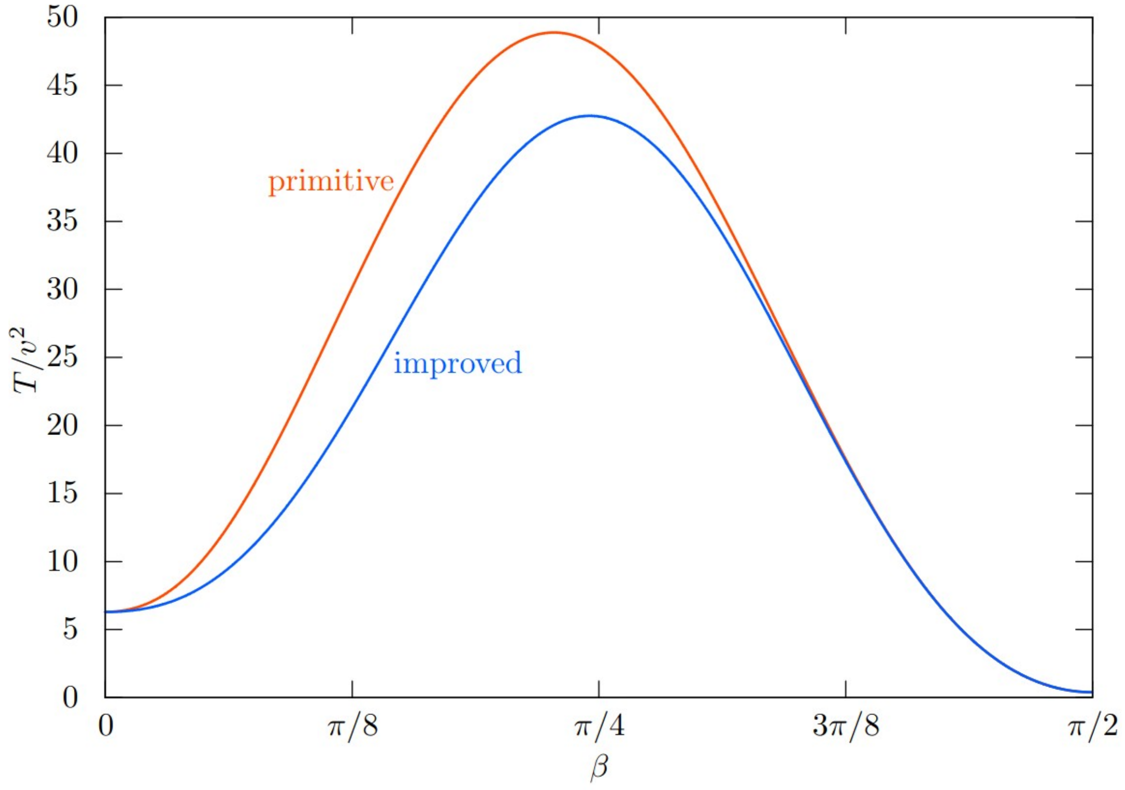
ϕ	Triplet	$\langle \phi^a \rangle = V \delta^{a3}$
h	Doublet	$\langle h \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$

Lagrangian:

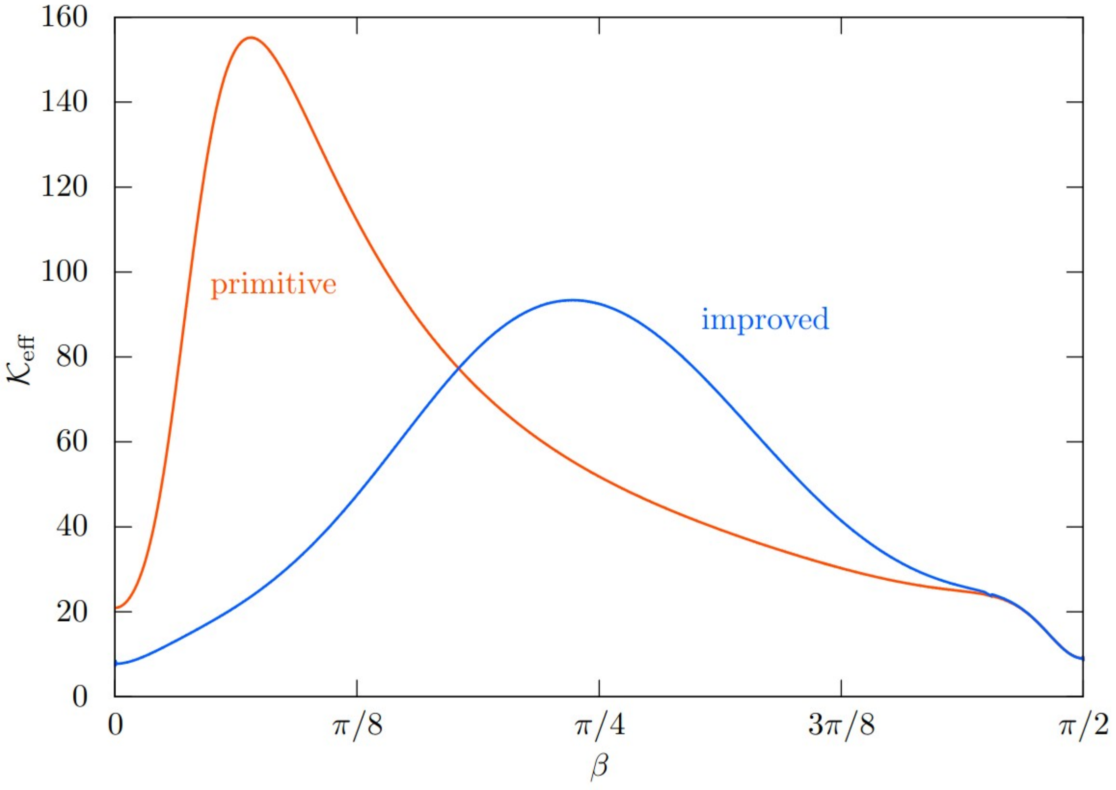
$$\mathcal{L} = -\frac{1}{2}(D_\mu \phi^a)(D^\mu \phi^a) - (D_\mu h)^\dagger (D^\mu h) - \frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} - V_{\text{Higgs}}(\phi, h)$$

$$V_{\text{Higgs}}(\phi, h) := \lambda\left(|h|^2 - v^2\right)^2 + \tilde{\lambda}\left(\phi^a \phi^a - V^2\right)^2 + \gamma \left| \left(\phi - \frac{V}{2}\right) h \right|^2$$

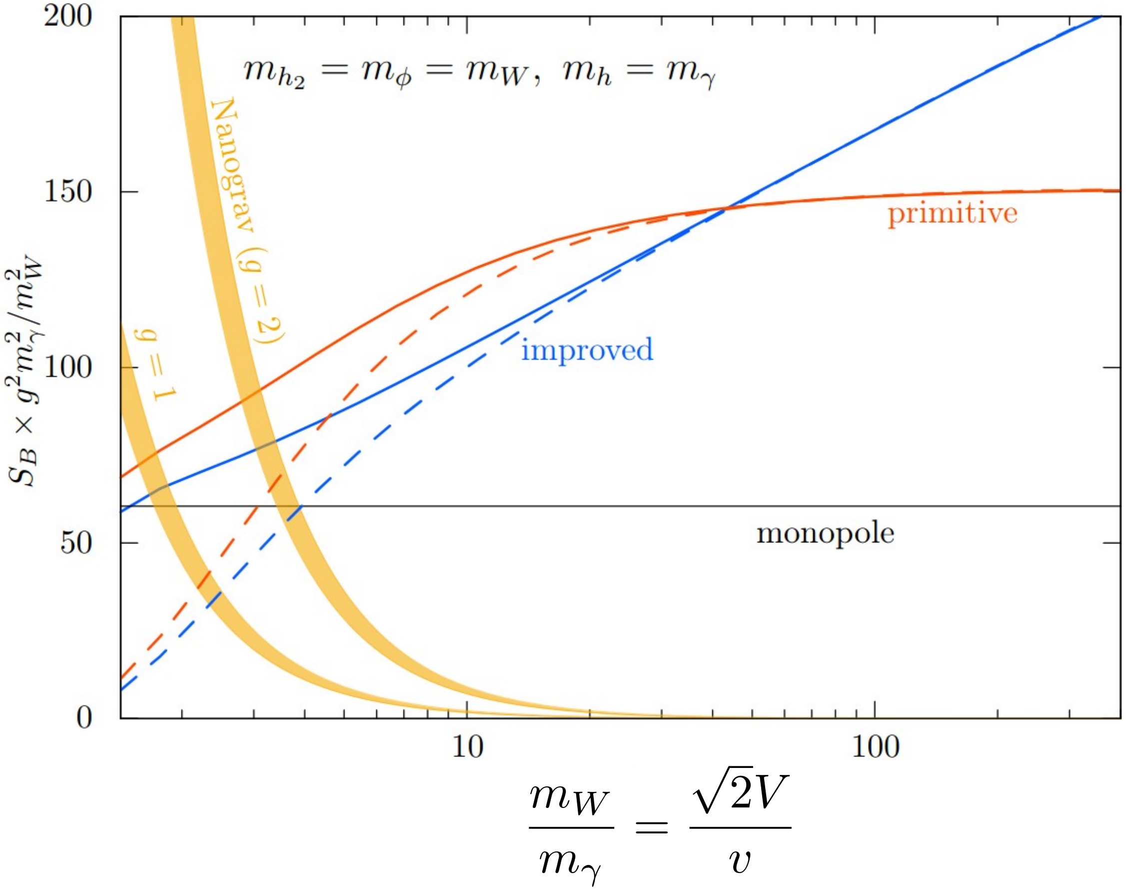
String Tension



Kinetic Term



Result



200
150
100
50
0
10
100
$S_B \times g^2 m_\gamma^2 / m_W^2$
$m_{h_2} = m_\phi = m_W, \ m_h = m_\gamma$
Nanograv ($g = 2$)
$g = 1$
primitive
improved
monopole
$\frac{m_W}{m_\gamma} = \frac{\sqrt{2}V}{v}$